Basic Topology – 2015

1MA179

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Lectures

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Drop by my office if you have any course related questions. If you want to ensure that I am there, send me an email first. If, for whatever reason, I am not around, and you have an urgent question on topological matters, you might be able to find help on Stack Exchange or Freenode IRC.

Course homepage

All documents handed out during the course, as well as some freely available literature, will be made available on the course homepage on Studentportalen. Studentportalen is also used to register for the course, sign up for exams, etc.

Moreover, in case anything happens to Studentportalen, I intend to keep a mirror of all available documents on my personal webpage.

Reading material

The official course literature is [M]:
Topology (2nd ed.), J. R. Munkres, Prentice Hall (2000)
of which we will cover roughly the first half. More precisely, material from chapter 1 partly forms background material that I will assume familiarity with, while other parts will be introduced when they enter the main story. We will cover more or less everything from chapters 2–4. In addition we will cover parts of chapters 5, 7, as well as a significant part of chapter 9. We will possibly cover material that cannot be found in the book; if so, any such material will be made available on the course homepage.

The course will closely resemble last year's edition, for which extensive course notes are available [F]. These will therefore also serve as a handy reference for what we do. Note that these notes contain a lot less "fluff" than the book of Munkres – this can be a good thing as it easier to go from theorem to theorem, but be aware that the fluff of Munkres is what contains most of the motivation for what we do.

Many other useful resources on the topic exist, as simply searching the web for "point-set topology" or "general topology" will show. If a particular point is unclear from the above references, it is not unlikely that one can find alternative descriptions online.

Finally, I have produced notes myself as we moved along; primarily to keep track of what material has been covered in the lectures. When our road through topology has deviated from the ones set up for us by the above references, this deviation will be included in these notes.

Form of teaching

The course consists of 20 events distributed into 16 lectures and 4 exercise sessions, and at the end of the course there is one lecture devoted to repetition and problem solving.

Way of learning

It is important to spend time with texts (the course book, lecture notes, other literature, ...) in order to go through and understand definitions, theorems, proofs, and examples. This, however, will not be enough. In addition it is important to sit down and solve exercises. That is the measure with which you can judge your understanding of the textbook material, and the most efficient way to gain familiarity with the plethora of definitions that we will introduce.

Exercises can be found in [M]. In addition, four problem sheets (partly overlapping with the exercises found in the aforementioned texts) will be made available at the end of my lecture notes. Solutions to *three of the problems* on each respective sheet can be handed in and will be corrected. The deadlines for handing in solutions are April 6th, April 23rd, May 6th and May 18th for exercise set number 1, 2, 3, and 4 respectively; optimally, solutions are handed in via email.

Exam

There will be a written final exam on Thursday, June 4th, 08:00—13:00, from which the total grade will be determined. At the exam, a number of exercises will be posed, the solution to each of which is worth a number of points $n \in \mathbb{N}$. If a solution to a given exercise is worth n points, then a partial solution is worth m points for some $m \in \{0, 1, \ldots, n\}$, depending on how partial the solution is.

The sum of the maximal number of possible points for will be 40, and to obtain a final grade of 3, 4, or 5 points, the sum of the points associated to the handed-in solutions will have to be 18, 25, or 32 points respectively.

If the course is taken as a PhD course, the only grading is whether or not the course is passed or failed. In this case, the threshold for passing is 23 points.

At the exam, no aids beyond pen/pencil and eraser are allowed. It is, however, allowed to use results from the lecture notes, both mine and Fjelstad's, as well as from the relevant chapters of [M]. If you use such a result, make sure to specify what that result says in general.

Lecture plan

The following is a rough outline of what have been covered in each of the lectures. For most lectures, there will be some overlap from previous lectures.

Date	Topics	[M]	[F]
Tu 24/3	Terminology and basic concepts from set theory; topological spaces, bases	§1, §3, §12, §13	1.1, 1.2, 1.3; 2.1
Th 26/3	More on bases, metric spaces, continuity	§18, §20, §21	2.2, 2.3
Fr 27/3	The subspace, poset, and order topologies, product topology	§14, §15, §16, §19	2.4, 2.5, 2.6
Mo 30/3	Interior, closure, density, separation: T_0, T_1, Hausdorff	§17, §20, §21	2.7, 2.8
Th 2/4	**Exercise session** (Set #1)	–	–
Mo 13/4	Sequences, limits, first countability, homeomorphisms, the n-sphere	§17, §20, §21, §18	2.9, 3.1, 3.2
We 15/4	Quotient topology	§22	3.3
Th 16/4	Connected spaces, connectedness of $\mathbb{R}$	§23, §24	4.1
Mo 20/4	Pathwise connectedness, connected components	§24, §25	4.2, 4.3
Th 23/4	**Exercise session** (Set #2)	–	–
Fr 24/4	Compactness, compactness of products	§26; §37	5.1, 5.2
Mo 27/4	Compactness in $\mathbb{R}^n$	§27 , §28	5.3
Tu 28/4	Locally compact spaces, one-point compactifications	§29	5.4
Tu 5/5	Countability and separation axioms, normal spaces, manifolds	§30, §31, §32, §36,	6.1, 6.2, 6.3, 7.1
We 6/5	**Exercise session** (Set #3)	–	–
Fr 8/5	Embeddings of manifolds, homotopy, path homotopy	§36, §51	7.2, 8.1
Mo 11/5	Properties of homotopy, the fundamental group	§51, §52	8.1, 8.2
Tu 12/5	The fundamental group of spheres	§53, §54	8.3
Mo 18/5	**Exercise session** (Set #4)	–	–
Th 21/5	Repetition and old exams	–	–
Th 4/6	Exam	–	–

References

[F] J. Fjelstad, *Notes on Topology, Lecture notes for Basic Topology 2014*

[M] J. R. Munkres, *Topology*, 2nd ed., Prentice Hall (2000)